SURFACES IN POSITIVE CHARACTERISTIC
MOSCOW, 4–8 APRIL 2016
ABSTRACTS

Alexander Duncan (University of South Carolina)
Finite subgroups of the plane Cremona group in positive characteristic

The plane Cremona group is the group of birational automorphisms of the projective plane. We consider finite subgroups of the plane Cremona group when the base field has positive characteristic. Following the same approach that has been so successful in characteristic zero, we use the equivariant minimal model program to reduce the question to studying automorphisms of rational surfaces.

In the first lecture, I will review the minimal model program for smooth surfaces, as well its equivariant cousins, emphasizing unusual behaviour in positive characteristic. I will then specialize to the case of rational surfaces and outline how one can use the theory to study the plane Cremona group. The remaining lectures will focus on the automorphisms of an important class of such surfaces: the del Pezzo surfaces of low degree.

In the second lecture, I will consider del Pezzo surfaces of degree 4: smooth complete intersections of two quadrics in 4-dimensional projective space. Here, characteristic 2 requires special care. In the third lecture, I consider automorphisms of cubic surfaces (degree 3). In the final lecture, we consider the cases of degree 1 and 2. Throughout, the characteristic 0 case will be reviewed, but the emphasis will be on new phenomena in positive characteristic.

Christian Liedtke (Technische Universität München)
Arithmetic Moduli of Enriques Surfaces

Using ideas of Cossec, Enriques, and Verra over the complex numbers, we show that all Enriques surfaces in arbitrary characteristic arise as quotients of complete intersections of three quadrics in projective 5-space by a finite flat group scheme of order 2. We use this to construct and to describe the local and global geometry of the moduli space of Enriques surfaces in arbitrary characteristic, as well as over the integers. In particular, this moduli space has two irreducible components in characteristic two, meeting along a divisor. Moreover these results also imply that all Enriques surfaces lift possibly ramified to characteristic zero.

Ronald van Luijk (Mathematisch Instituut Leiden)
The arithmetic of del Pezzo surfaces

The geometry of smooth cubic surfaces in projective three-space $\mathbb{P}^3$ is well understood. Over an algebraically closed field, they are all isomorphic to the projective plane $\mathbb{P}^2$ blown up in six points in general position; they contain 27 lines that can be expressed in terms of these six points. Over the field of rational numbers, however, a cubic surface may not even contain any rational points. As soon as it does, though, it is unirational by a Theorem of Kollár, which completed a result by Manin.

This series of lectures is about the arithmetic of del Pezzo surfaces, which over an algebraic closure of the ground field are isomorphic to either $\mathbb{P}^1 \times \mathbb{P}^1$ or $\mathbb{P}^2$ blown up in r points in general position, with $0 \leq r \leq 8$. For small r, the arithmetic of del Pezzo surfaces is fairly simple: as soon as they have a rational point, they are unirational, or even rational. We will look at the proof of this celebrated result by Manin. We will also analyse what can be said for the more diificult cases $r = 7$ and $r = 8$. Most results will hold for arbitrary fields, but we will give special attention to number fields and finite fields.

Sergey Rybakov (IITP and HSE)
Zeta functions of algebraic surfaces over finite fields

The Honda–Tate theory gives a classification of zeta functions of abelian varieties over finite fields. I will give a short introduction to this theory and then apply it to a classification of zeta functions of Kummer and bielliptic surfaces.

Lenny Taelman (University of Amsterdam)
Zeta functions of $K3$ surfaces over finite fields

The zeta function of a $K3$ surface over a finite field satisfies a number of well-known (archimedean and l-adic) and a number of less obvious (p-adic) constraints coming from their étale and p-adic cohomology. We consider the converse question, in the style of the Honda–Tate theorem: given a finite field and a function satisfying these constraints, is it the zeta function of a $K3$ surface over this finite field? We give a partial answer to this question, making heavily use of the theory of moduli of $K3$ surfaces over the complex numbers.

The first lecture will be a general overview/introduction into the theory of $K3$ surfaces, with an emphasis on moduli over the complex numbers. No prior knowledge of $K3$ surfaces will be assumed. In the second lecture I will explain how to use the complex theory to produce $K3$ surfaces over finite fields, and how to partially answer the above question.

Andrey Trepalin (IITP and HSE)
Zeta functions of geometrically rational surfaces

We consider a geometrically rational minimal surface X over a finite field k. The zeta function of such surface is determined by the image of the Galois group $\Gamma = \mathrm{Gal}(\overline{k}/k)$ in the automorphism group of the Picard lattice $\mathrm{Pic}(\overline{X})$. For each cyclic subgroup C of $\mathrm{Aut}\big(\mathrm{Pic}(\overline{X})\big)$ where X is a conic bundle or a del Pezzo surface of degree at least 2, and $\mathrm{Pic}(\overline{X})^C$ is isomorphic to $\mathbb{Z}^2$ or $\mathbb{Z}$ respectively, we show that there exists a geometrically rational surface X such that C is conjugate in $\mathrm{Aut}\big(\mathrm{Pic}(\overline{X})\big)$ to Γ (under mild assumptions on the ground field in some cases).

Christian Liedtke (Technische Universität München)
Good Reduction of K3 surfaces (Friday seminar talk)

By a classical theorem of Serre and Tate, extending previous results of Néron, Ogg, and Shafarevich, an Abelian variety over the field of fractions K of a local Henselian DVR has good reduction if and only if the Galois action on its first l-adic cohomology is unramified ("no monodromy"). We show that if the Galois action on second l-adic cohomology of a $K3$ surface over K is unramified, then the surface admits an "RDP model", and good reduction (that is, a smooth model) after a finite and unramified extension. (Standing assumption: potential semi-stable reduction.) Moreover, we give examples where such an unramified extension is really needed. This is joint work with Yuya Matsumoto.