**Lectures by Jean-Michel Bismut**

Index theorem and the hypoelliptic Laplacian

I will explain some of the developments which took place in index theory these last twenty years.

I will review the local index theorem for Dirac operators, including the proof of the local families index theorem for families of Dirac operators using Quillen's superconnections.

I will explain the compatibility of these results to complex and algebraic geometry. In particular I will explain some of the results I proved with Berthomieu, Gillet, Köhler, Lebeau, Soulé on Quillen metrics on determinants of direct images.

The Fourier transform aspects of index theory will be emphasized. This will motivate the introduction of the hypoelliptic Laplacian, which is supposed to provide a natural interpolation between the classical Laplacian and the geodesic flow. I will show how to obtain a natural construction of the hypoelliptic Laplacian in de Rham theory.

I will explain how the evaluation of semisimple orbital integrals via the hypoelliptic Laplacian can be understood as a Riemann-Roch formula. In this context, the Fourier transform aspect of index theory becomes dominant. Time permitting, I will describe some applications of the hypoelliptic Laplacian in complex geometry.

# Bibliography

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[3] J.-M. Bismut. *Hypoelliptic Laplacian and orbital integrals*, volume 177 of *Annals of Mathematics Studies.* Princeton University Press, Princeton, NJ, 2011.

[4] J.-M. Bismut. Index theory and the hypoelliptic Laplacian. In *Metric and differential geometry*, number 297 in Progress in Mathematics, pages 181–232. Birkhäuser/Springer, Basel, 2012.

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