ZETA FUNCTIONS COMING FROM GEOMETRY

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Abstract and some references

1. Classical zeta functions : Riemann zeta function, Dirichlet L-functions, prime number theorem and arithmetic progression theorem; Dedekind zeta functions and Artin L-functions, Chebotarev theorem.

(1) S. Lang, *Algebraic number theory*, Addison-Wesley, 1970.
(2) J. Tate, *Fourier Analysis in Number fields, Hecke's Zeta-Functions*, 1950.

2. Zeta functions from algebraic geometry : Weil zeta function (for a variety over a finite field); Hasse-Weil L-functions (for a variety over a number field); L-function associated to a Galois representation or a modular form.

(1) R. Hartshone, *Algebraic geometry (Appendix C)*, Springer-Verlag, 1983.
(2) F. Diamond, J. Shurman, *Introduction to modular forms*, Springer, 2008.
(3) J-P. Serre, *Facteurs locaux des fonctions zêta des variétés algébriques (définitions et conjectures)*, Séminaire DPP, 1969/70, exposé 19.
(4) J-P. Serre, *Zeta and L-functions*, in Arithmetic Algebraic Geometry, New York: Harper and Row, 1965.
(5) J. Tate, *Algebraic cycles and poles of zeta functions*, in Arithmetic Algebraic Geometry , New York: Harper and Row, 1965.

3. Analytic theory of zeta functions : Analytic continuation and functional equations; Analytic estimates; Generalized Riemann hypothesis.

(1) H. Davenport, *Mutiplicative Number Theory*, GTM 74, Springer, 1980.
(2) H. Iwaniec, E. Kowalski, *Analytic number theory.* AMS, 2004.

4. Special values of zeta functions : Class number formula; Birch and Swinnerton Dyer conjecture; Brauer-Siegel type theorems. If time permits, we will mention other conjectures on special values : Deligne, Beilinson, etc.

(1) A. Beilinson, *Higher regulators and values of L-functions* J. Soviet Math. **30**, 2036–2070, 1985.
(2) S. Bloch, K. Kato, *L-Functions and Tamagawa Numbers of Motives, in* The Grothendieck Festschrift, Vol. 1 , Birkhäuser, 1990.
(3) P. Deligne, *Valeurs de fonctions L et périodes d'intégrales.* Symp. Pure Math. A.M.S. 33, 313–346, 1979.
(4) B. Gross, D. Zagier, *Heegner points and derivatives of L-series*, Inventiones Mat. **84**, 225–320, 1986.
(5) S. Lichtenbaum, *Zeta functions of varieties over finite fields at $s = 1$.* Arithmetic and geometry, Vol. I, Birkhäuser, 1983.
(6) J. Tate, *On the Birch and Swinnerton-Dyer conjecture and a geometric analog*, Sém. Bourbaki exp. **306**, 1965/66.

I will use (some part of the following) Course notes :
http://www.math.jussieu.fr/~hindry/Notes_rev_Brasilia.pdf