WORKSHOP ON PROJECTIVE ALGEBRAIC GEOMETRY
MOSCOW, 7–12 SEPTEMBER 2015
ABSTRACTS

Roland Abuaf (Imperial College London)
Hyper-Kaehler categories

In this talk I will introduce the notion of hyper-Kaehler categories and study some of their basic properties. I will focus in particular on construction techniques. I hope to convey the idea that hyper-Kaehler geometry is best studied from the non-commutative point of view.

Lucian Badescu (Genoa University)
Infinitesimal extensions of rank two vector bundles on submanifolds of small codimension

Let X be a submanifold of dimension n of the complex projective space $\mathbb{P}^N$ $(n < N)$, and let E be a vector bundle of rank two on X . If $n \geqslant \frac{N+3}{2} \geqslant 4$ we prove a geometric criterion for the existence of an extension of E to a vector bundle on the first order infinitesimal neighborhood of X in $\mathbb{P}^N$ in terms of the splitting of the normal bundle sequence of $Y \subset X \subset \mathbb{P}^N$, where Y is the zero locus of a general section of a high twist of E. In the last section we show that the universal quotient vector bundle on the Grassmann variety $\mathrm{Gr}(k, m)$ of k-dimensional linear subspaces of $\mathbb{P}^m$, with $m \geqslant 3$ and $1 \leqslant k \leqslant m-2$ (i.e. with $\mathrm{Gr}(k, m)$ not a projective space), embedded in any projective space $\mathbb{P}^N$, does not extend to the first infinitesimal neighborhood of $\mathrm{Gr}(k, m)$ in $\mathbb{P}^N$ as a vector bundle.

Fedor Bogomolov (Courant Institute and HSE)
Symmetric tensors and the geometry of subvarieties of $\mathbb{P}^N$

Ivan Cheltsov (University of Edinburgh and HSE)
Rational nodal Fano threefolds

I will describe the rationality problem for nodal quartic double solids and nodal quartic threefolds. In particular, I will prove that nodal quartic double solids with at most six singular points are irrational, and nodal quartic double solids with at least eleven singular points are rational. I will show how to use two classical constructions of Todd to prove rationality of two nodal quartic threefolds in the family studied by Beauville, who proved the irrationality of all other quartic threefolds in this family (except Burkhardt and Igusa quartics that are also rational). This is a joint work with Victor Przyjalkowski and Constantin Shramov.

Olivier Debarre (École Normale Supérieure)

Cubic hypersurfaces over finite fields

We examine when a cubic hypersurface defined over a finite field contains lines and study the zeta function of its variety of lines. This is joint work with A. Laface and X. Roulleau.

Sergey Finashin (Middle East Technical University)

Real deformations of Cayley octads and their links

Cayley octads are 8-configurations in $\mathbb{P}^3$ that are complete intersections of three quadrics (i. e., the base locus of a net of quadrics). The locus formed by singular quadrics of the net form a Hessian (or spectral) quartic endowed with an even theta characteristic. An octad is called regular if this quartic is non-singular. Using this correspondence one can obtain a deformation classification of real regular Cayley octads which will be discussed. Namely, I will present the 8 deformation classes of maximal real regular Cayley octads and discuss their monodromy groups and degenerations. I will relate also Cayley octads with the 14 real deformation classes of 7-configurations (Aronhold sets) which appear to be their links.

Baohua Fu (Chinese Academy of Sciences)

On special birational transformations

A birational transformation $f\colon \mathbb{P}^n \dashrightarrow Z$, where Z is a nonsingular variety of Picard number 1, is called a special birational transformation of type (a, b) if f is given by a linear system of degree a, its inverse is given by a linear system of degree b and the base locus $S \subset \mathbb{P}^n$ is irreducible and nonsingular. I'll report a joint work with Jun-Muk Hwang on the classification of special birational transformations of type $(2, 1)$.

Sergey Gorchinskiy (Steklov Mathematical Institute and HSE)
Categorical measures for varieties with finite group actions

The talk is based on a common work with D. Bergh, M. Larsen, and V. Lunts. Given a variety with a finite group action, we compare categorical measures of the corresponding quotient stack and the extended quotient. Using weak factorization for orbifolds, we show that for a wide range of cases, these two measures coincide, which implies, in particular, a conjecture of Galkin and Shinder on categorical and motivic zeta-functions of varieties. We provide examples showing that in general, these two measures are not equal, which gives a counterexample to a conjecture of Polishchuk and van den Bergh on semi-orthogonal decompositions of equivariant derived categories.

Robin Hartshorne (University of California at Berkeley)
Algebraic Space Curves: old results and open problems

I will review the question of degree and genus of curves in $\mathbb{P}^3$ and their relation to vector bundles on $\mathbb{P}^3$. Then I will discuss the Halphen problem, to determine the maximum genus of a curve of degree d not contained in any surface of degree $s-1$. While parts of this problem have been known for some time, there is an important piece which is still an open question. Also, I will discuss families of curves in $\mathbb{P}^3$: the Hilbert scheme, its irreducible components, and questions of connectedness. Here there are many open questions.

Grzegorz Kapustka (Uniwersytet Jagielloński)
Twenty incident planes and hyperkaehler geometry

Answering to a problem of Morin and O'Grady we construct a complete family of 20 incident planes in $\mathbb{P}^5$ (this is the maximal cardinality of a complete finite family of incident planes). The construction is related to the geometry of the hyperkahler fourfold being the Hilbert scheme of two points of the "most algebraic" $K3$ surface considered by Vinberg. This is a joint work with B. van Geemen, M. Donten-Bury, M. Kapustka, and J. Wisniewski.

Michal Kapustka (Uniwersytet Jagielloński)
EPW cubes

I will report on a joint work with A. Iliev, G. Kapustka, and K. Ranestad. We construct a new 20-dimensional family of projective 6-dimensional irreducible holomorphic symplectic manifolds. The elements of this family are deformation equivalent with the Hilbert scheme of three points on a $K3$ surface. They are constructed as natural double covers of special codimension 3 subvarieties of the Grassmanian $\mathrm{Gr}(3,6)$. These codimension 3 subvarieties are defined as Lagrangian degeneracy loci and can be seen as generalizations of EPW sextics, we call them the EPW cubes.

Victor Kulikov (Steklov Mathematical Institute and HSE)
On some special actions of the symmetric group $\mathfrak{S}_4$ on $K3$ surfaces

In the talk, I'll consider the actions of the symmetric group $\mathfrak{S}_4$ on $K3$ surfaces X having the following properties:

(1) the alternating group $\mathfrak{A}_4 \subset \mathfrak{S}_4$ acts symplectically on X;

(2) there exists an equivariant bi-rational contraction $\overline{c}\colon X \to \overline{X}$ to a $K3$ surface $\overline{X}$ with ADE-singularities such that $\overline{X}/\mathfrak{S}_4 \simeq \mathbb{P}^2$.

I'll show that up to equivariant deformations there exist exactly 15 such actions and these actions can be realized as the actions of the Galois group on the Galois normal closure of the dualizing coverings of the projective plane associated with rational quartics having no singularities of types A_4, A_6 and E_6.

Alexander Kuznetsov (Steklov Mathematical Institute and HSE)
Birational isomorphisms of Gushel–Mukai varieties

A Gushel–Mukai variety is a prime Fano variety of coindex 3 and degree 10. These varieties can be explicitly described as either hyperquadric sections of linear sections of $\mathrm{Gr}(2,5)$ or as double covers of linear sections of $\mathrm{Gr}(2,5)$ ramified in a hyperquadric. In the talk I will discuss birational geometry of these varieties. In dimensions 5 and 6 these varieties are rational and in dimensions 3 and 4 two such varieties are birational as soon as their associated EPW sextics are isomorphic or projectively dual. This is a joint work with Olivier Debarre.

Sijong Kwak (Korea Advanced Institute of Science and Technology)
Syzygies and classification of quadratic and cubic projective schemes

For quadratic or cubic projective schemes, we give some upper bound on the generators and their relations generalizing the results of Castelnuovo and Fano. We introduce various methods on these problems and further questions to audience with modest backgrounds.

Angelo Lopez (Università degli Studi Roma Tre)
Positivity properties and stable base loci of cycles

Similarly to the case of divisors, one can define, on a given variety, the usual cones of effective, pseudoeffective, big and nef k-dimensional cycles (modulo numerical equivalence). Perhaps what is missing is a good notion of "positivity", as examples of Debarre–Ein–Lazarsfeld–Voisin and Ottem show. In the talk we will introduce augmented and restricted stable base loci for cycles. This leads to a new positivity property.

Serge Lvovski (HSE)
On projections of plane curves and Chisini's conjecture

This is a joint work with Yu. Burman.

A classical conjecture of Chisini claims that ramified coverings of the projective plane are uniquely determined by their branch loci. An amusing application of Picard–Lefschetz theory allows us to prove that Chisini's conjecture is true for surfaces ramified over duals to general nodal curves of degree at least 3, except for duals to smooth cubics. This strengthens a result of Victor Kulikov.

Laurent Manivel (Institut de Mathématiques de Marseille)
Hyperkaehler manifolds and the Tits-Freudenthal magic square

The Tits–Freudenthal magic square was discovered in the 1950's as a way to construct simple Lie algebras, most notably the exceptional ones, from a pair of normed algebras. Several types of geometries are related to the magic square, including the series of Severi varieties once classified by F. Zak. I will explain how to construct hyper-Kaehler varieties from the magic square.

John Christian Ottem (University of Cambridge)
Hypersurfaces in products of projective spaces

Hypersurfaces in multiprojective spaces are defined by a multigraded polynomial. Despite their apparent simplicity, their birational geometry can be quite complicated. For example, they can have infinite birational automorphism groups. We give a complete picture of their birational geometry and compute their cones of effective, movable and nef divisors. As an application, we give a negative answer to a question of Cascini and Gongyo and discuss a possible relation to a conjecture of Keel.

Christian Peskine (IMJ-PRG, Paris 6)
Projecting a complete intersection of $k+1$ quadrics from an interior $\mathbb{P}^k$. Graph and Fano variety of lines.

Project a quadric surface from one of its point. The graph is the blowing up of a plane in two points and the Fano variety of lines of the quadric is isomorphic to the normalization of the variety of lines (in the plane) passing through any of the two points.

Project a complete intersection of two quadrics $X \subset \mathbb{P}^5$ from a line $L \subset X$. The graph is the blowing up of $\mathbb{P}^3$ along a genus 2 and degree 5 curve S. The blowing up of the Fano variety of lines (of X) at the point $\{L\}$ is isomorphic to the normalization of the variety of 2-secant lines to S. In this isomorphism, the curve of special lines of X is isomorphic to the curve of stationary secant lines to S.

These well known examples are special cases of a general construction.

Project a (general enough) complete intersection $X \subset \mathbb{P}^N$ of $k+1$ quadrics, from a linear space $\Delta = \mathbb{P}^k \subset X$. The graph of the projection is the blowing up of $\mathbb{P}^{N-k-1}$ along a determinental subvariety $S \subset \mathbb{P}^{N-k-1}$, cut out by the maximal minors of a $(k+1) \times (k+2)$ matrix.

In a common work with Dan Avritzer, we study this graph and interpret it in terms of the linear spaces contained in X. In particular we check easily that the open scheme of the Fano variety of lines in X formed by lines disjoint from Δ is isomorphic to an open set of the variety of $(k+1)$-secant lines to S. Furthermore we conjecture, under certain hypotheses of generality, and prove in some cases, that the blowing up of the Fano variety along the subvariety of lines intersecting Δ is the normalization of the variety of $(k+1)$-secant lines to S.

Yuri Prokhorov (Steklov Mathematical Institute and HSE)
Singular Fano threefolds of type V_{22}

I will discuss mild degenerations of smooth prime Fano threefolds of type V_{22}.

Kristian Ranestad (Universitetet i Oslo)
On cactus varieties of homogeneous forms

The study of higher secant varieties has a long history, and has only recently been supplemented by the corresponding study of the union of $(r-1)$-dimensional linear spaces that intersect a smooth projective variety X in a scheme of length at least r. The closure of the latter is called the r-th cactus variety of X. I shall discuss upper bounds for the dimension of the cactus variety, when X is the 3-uple embedding of projective n-space. This is a report on ongoing work with Bernardi, Jelisiejew and Marques.

Andrey Trepalin (Institute for Information Transmission Problems and HSE)
Minimal cubic surfaces over finite fields

Let us consider a cubic surface over finite field $\mathbb{F}_q$ of characteristic greater than 3. It is well-known that there are five possibilities for the image of the Galois group $\mathrm{Gal}(\overline{\mathbb{F}}_q/\mathbb{F}_q)$ in the Weyl group $W(E_6)$ such that the cubic surface is minimal. We explicitly construct examples of minimal cubic surfaces for each of those possibilities in case $q = 3k+1$ and some other cases.

Fyodor Zak (CEMI)
Convex bodies, dual varieties, and osculating spaces

Basing on contributions of several mathematicians among whom there are five participants of the workshop, we consider the analogy between supporting hyperplanes of convex bodies and tangent hyperplanes to nonsingular projective varieties. We prove an analog of the Anderson–Klee theorem for projectively dual varieties and apply it to study the osculating behavior of smooth projective varieties.