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## Some curiosities on Spin(9) and the sphere $S^{15}$

### Abstract

Although holonomy Spin(9) is only possible for the two 16-dimensional symmetric spaces  $\mathbb{O}P^2$  and  $\mathbb{O}H^2$ , weakened holonomy Spin(9) conditions have been proposed and studied, in particular by Th. Friedrich. A basic problem is to have a simple algebraic formula for the canonical 8-form  $\Phi_{\text{Spin}(9)}$ , similar to the usual definition of the quaternionic 4-form  $\Phi_{\text{Sp}(n)\cdot\text{Sp}(1)} = \omega_I^2 + \omega_J^2 + \omega_K^2$ , witten in terms of local compatible almost hypercomplex structures  $(I, J, K)$ .

In the talk, a simple formula for  $\Phi_{\text{Spin}(9)}$  is presented, discussing a family of local almost hypercomplex structures associated with a Spin(9) manifold  $M^{16}$ . Some of these complex structures, now on model spaces  $\mathbb{R}^{16^q}$ , are then used to give an approach through Spin(9) to the very classical problem of writing down a maximal system of tangent vector fields on spheres  $S^{N-1} \subset \mathbb{R}^N$ . If time permits, some properties of manifolds equipped with a locally conformal parallel Spin(9) metric will be also discussed.

The talk will report on joint work with M. Parton and V. Vuletescu.

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