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Some curiosities on Spin(9) and the sphere $S^{15}$

Abstract

Although holonomy Spin(9) is only possible for the two 16-dimensional symmetric spaces $O^2 P^2$ and $O^2 H^2$, weakened holonomy Spin(9) conditions have been proposed and studied, in particular by Th. Friedrich. A basic problem is to have a simple algebraic formula for the canonical 8-form $\Phi_{\text{Spin}(9)}$, similar to the usual definition of the quaternionic 4-form $\Phi_{\text{Sp}(n)\cdot\text{Sp}(1)} = \omega_I^2 + \omega_J^2 + \omega_K^2$, written in terms of local compatible almost hypercomplex structures $(I, J, K)$.

In the talk, a simple formula for $\Phi_{\text{Spin}(9)}$ is presented, discussing a family of local almost hypercomplex structures associated with a Spin(9) manifold $M^{16}$. Some of these complex structures, now on model spaces $\mathbb{R}^{16q}$, are then used to give an approach through Spin(9) to the very classical problem of writing down a maximal system of tangent vector fields on spheres $S^{N-1} \subset \mathbb{R}^N$. If time permits, some properties of manifolds equipped with a locally conformal parallel Spin(9) metric will be also discussed.

The talk will report on joint work with M. Parton and V. Vuletescu.

References

[8] M. Parton, P. Piccinni, Spheres with more than 7 vector fields: all the fault of Spin(9), arXiv: 1107.0462.

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