Polar actions on symmetric spaces

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Abstract

An isometric action of a connected Lie group H on a Riemannian manifold M is called polar if there exists a connected closed submanifold Σ of M such that Σ meets each orbit of the action and intersects it orthogonally. An elementary example of a polar action comes from the standard representation of SO_n on $\mathbb{R}^n$. Further examples of polar actions can be constructed from Riemannian symmetric spaces. Let $M = G/K$ be a Riemannian symmetric space and denote by o a fixed point of the K-action on M. Then the isotropy representation $\pi : K \to O(T_oM)$ of K on the tangent space T_oM of M at o induces a polar action. Dadok established in 1985 a remarkable, and mysterious, relation between polar actions on Euclidean spaces and Riemannian symmetric spaces. He proved that for every polar action on $\mathbb{R}^n$ there exists a Riemannian symmetric space $M = G/K$ with $\dim M = n$ such that the orbits of the action on $\mathbb{R}^n$ and the orbits of the K-action on T_oM are the same via a suitable isomorphism $\mathbb{R}^n \to T_oM$.

Soon afterwards an attempt was made to classify polar actions on symmetric spaces. For irreducible symmetric spaces of compact type the final step for a complete classification appears to have been just completed by Kollross using yet unpublished work of Lytchak on polar foliations. In the talk I want to focus on symmetric spaces of noncompact type. For actions of reductive groups one can use the concept of duality between symmetric spaces of compact type and of noncompact type. However, new examples and phenomena arise from the geometry induced by actions of parabolic subgroups, for which there is no analogon in the compact case. I plan to discuss the main difficulties one encounters here and some partial solutions. The only complete classification known so far has just been obtained in joint work with José Carlos Díaz-Ramos for the complex hyperbolic plane.