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ON GUAN'S EXAMPLES OF SIMPLY CONNECTED NON-KAHLER COMPACT COMPLEX MANIFOLDS

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Abstract. The article contains a construction of compact complex symplectic manifolds which are simply connected and don't admit Kahler structure. The manifolds obtained are essentially the same as were constructed by Guan, but the construction itself is geometrically more transparent.

0. Introduction. The first examples of compact complex symplectic manifolds which are non-Kahler were constructed by D. Guan in a series of preprints [3], [4], [5]. In particular, he constructed simply connected manifolds with the above property, thus disproving the conjecture of A. Todorov [7].

Guan's results indicate the importance of finding simple criteria which distinguish Kahler and non-Kahler compact complex symplectic manifolds. The failure of Todorov's conjecture notwithstanding, it still may be the case that there is a purely topological criterion which does this. For example, it might turn out that the multiplication structure on the second cohomology group and the triviality of secondary cohomological operations distinguish Kahler and non-Kahler manifolds of the above type.

The main idea of Guan's first approach is to apply the Beauville-Fujiki construction via symmetric powers ([1], [2]) to a non-Kahler Kodaira surface [6]. Guan's construction is purely geometrical, but the proof of its consistency is based on some calculations. The aim of this article is to provide a geometrical explanation of the calculations involved and arithmetical constraints imposed on the initial data in Guan's construction.

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1. Kodaira surfaces. The class of Kodaira surfaces is described in [6]. Here I will present a summary of this description, omitting the proofs.

Any Kodaira surface can be obtained in the following way: Let us take a line bundle L over an elliptic curve E with $c_1(L) = m \neq 0$. Denote the complement of zero section in L by L^*. The group C^* acts freely and fiberwise on L^*. The

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nonzero element of the Lie algebra of C^* defines a nondegenerate holomorphic field e_L on L^*. There is a C^* invariant nondegenerate holomorphic (2,0)-form w on L^* defined by the following property: $(w(e_L)) = \omega_1$. Here ω_1 is a holomorphic (1,0)-form induced from the projection of L^* on elliptic curve E. Let $Z \in C^*$ be a discrete cocompact subgroup. Define the Kodaira surface S as a quotient L^*/Z. It is a compact surface S which is a fibration with elliptic fiber $C^*/Z = E_L$ over E. The form w defines a holomorphic nondegenerate form on S which we will denote also by w.

There is a natural holomorphic action of E_L on S. Moreover the canonical class of S is trivial, and the global holomorphic form w on S is invariant under the action of E_L.

Topologically, S has a structure of a principal fibration over T^3 with S^1 as a fiber. It is a principal fibration. The bundle L is associated to S^1 bundle over E, which defines a homogeneous norm function on L and L^*. Thus we obtain a projection, $L^*/Z \to R^*/Z = S^1_0$. Therefore S has a presentation as a product of a nontrivial principal S^1-bundle over E and a circle S^1_0. The product $E \times S^1_0$ will be denoted as T^3. The circle S^1 above is a subgroup of E_L.

Thus S is a principal holomorphic E_L fibration over E with a projection p_E. It is also a principal S^1 fibration over T^3 with a projection $p_T = p_s p_T$ where p_s: $T^3 \to E$ contracts S^1_0.

The first homology group, $H_1(S, Z)$, is isomorphic to $Z^3 + Z_m$ and the fundamental group of S is a cyclic central extension of $H_1(S, Z)$. In fact the surface S is completely described by the elliptic curves E, E_L and $m = c_1(L) > 0$. The latter will be also called the first Chern class of the elliptic fibration and will be denoted as $c_1(E^S_L)$ or simply c_1.

Apart from this we shall need the following simple facts about Kodaira surfaces:

1. The only compact complex curves on S are the fibers of the projection p_E.
2. Any smooth compact complex surface V with a surjective projection onto S is non-Kahler. In fact, such a surface is also an elliptic fibration with the first cohomology group $H_1(V, R)$ of odd rank.

We can construct a similar manifold of higher dimension starting from a set of n line bundles L_i over E with the first Chern classes $c_1(L_i) = c^i_1$. Let H be a fiber product of L^*_i over E. We can factorize H by a natural action of Z^n and obtain a compact complex manifold $S_n = H/Z^n$. The manifold S_n is naturally fibered over E with a fiber T which is a product of n copies of E_L.

LEMMA 1.1. *Assume that the elements L_i define a sublattice of rank one in PicE and at least one of $c^i_1 \neq 0$. Then the manifold S_n has a finite nonramified covering S^c_n which is a product of a Kodaira surface S and a torus T', where T' is isogenious to the product on $n - 1$ copies of E_L.*

Proof. A natural fiberwise map n_i: $L_i \to L^n_i$ induces a nonramified covering n_i: $L^*_i \to L^{n*}_i$. The latter is C^* equivariant. Thus we can assume that all L_i

are isomorphic via a translation on E by taking an appropriate collection of n_i. Therefore, L_i becomes a trivial bundle being lifted on L_1^* and the trivialization is compatible with C^*-actions on L_i and L_1^*. Therefore S_n has a nonramified covering which is a direct product of L_1^*/Z and a torus which is a product of the curves E_L.

Remark 1.2. The situation is quite different if the bundles L_i define independent classes in the group $PicE$. All proper complex subvarieties of S_n are contained in the fibers T' in the opposite case when the bundles L_i correspond to linearly independent classes in $PicE$.

2. Desingularization of a symmetric power of the surface. Guan's construction begins with a surface S with $c_1(L) = m > 2$. Suppose $r > 2$ is a number dividing m. From this data we construct a complex symplectic manifold of dimension $2r - 2$.

Consider the r-symmetric power $S^r S$ of S. It is a singular complex variety which has a standard Douady desingularization, $S[r]$. The points of $S[r]$ correspond to zero-dimensional subschemes of S having length r.

PROPOSITION 2.1. *The complex manifold $S[r]$ has the following properties:*

(1) *It is a complex compact manifold and carries a nondegenerate holomorphic form.*

(2) *It has an abelian fundamental group equal to $H_1(S, Z)$.*

(3) *It is fibered over T^3 with smooth irreducible fibers.*

(4) *The natural projection p_t: $S[r] \to T^3$ induces an isomorphism of $\pi_1(S[r])/Z_m$ and $\pi_1(T^3)$.*

Proof.

(1) $S[r]$ is a canonical resolution of $S^r S$ (see [1]) and hence it has a nondegenerate holomorphic (2,0)-form w. The latter is induced from the holomorphic (2,0)-form on S by the natural symmetrization.

(2) The map p_T: $S \to T^3$ induces a map p_t: $S[r] \to T^3$ since T^3 has a structure of an abelian group. The surface S fibers over T^3 and a locality of the desingularization implies that the same holds for the projection p_t of $S[r]$. Therefore, $S[r]$ is fibered over T^3 with a smooth fiber.

Similarly there exists the projection p_e: $S[r] \to E$ and $p_e = p_s p_t$.

(3) Since $r > 1$ the fundamental group $\pi_1(S^r S)$ coincides with the abelianization of $\pi_1(S)$. It also coincides with the fundamental group of the smooth part of $S^r S$. Hence $\pi_1(S[r]) = \pi_1(S^r S)$.

(4) The natural map of $\pi_1(S)$ into $\pi_1(S^r S)$ is defined by choosing any coordinate map of S into S^r and then applying symmetrization. Hence, the map p_{t*}: $\pi_1(S[r]) \to \pi_1(T^3)$ coincides with the abelianization of the corresponding map for S.

There is a natural diagonal action of E_L on $S[r]$. This action is locally free since the action of E_L on S is free. It is also a fiberwise action for p_e since it is fiberwise on S. For a similar reason, the action of $S^1 \subset E_L$ on $S[r]$ is fiberwise for the projection p_T.

Let us denote $p_t^{-1}(0)$ by M_r and $p_e^{-1}(0)$ by W_r. The latter is a smooth complex manifold with a locally free action of E_L. The holomorphic vector field e_L tangent to the orbits of E_L is dual to the holomorphic one form induced from E under p_e. Therefore, e_L coincides with the kernel of the restriction of w to W_r. Similarly, the tangent field e_s of the action of S^1 coincides with the kernel of w on M_r.

3. Construction. Let us take the quotient W_r/E_L. It is a complex compact variety with quotient singularities. The form w induces a holomorphic (2,0)-form w' on the part of W_r/E_L corresponding to the free orbits of E_L.

There exists a compact complex manifold Q with the surjective map $p_q\colon\ Q \to W_r/E_L$ of the degree r^2 that $p_q^* w'$ is a nonsingular closed holomorphic (2.0)-form on Q. The proof of the existence of such a manifold is the most nontrivial part of Guan's construction. Here we shall use a different argument to prove the existence of Q.

We begin with the manifold M_r which is a real manifold of dimension $4r-3$. Consider the quotient $R_r = M_r/S^1$. We shall show that this is a compact complex variety with quotient singularitues and a map $p_r\colon\ R_r \to W_r/E_L$ of degree r. The manifold Q can be obtained as a cyclic r-covering of R_r which is ramified only at the singular points of R_r.

On the local manifold transversal to the orbit of S^1 there is a natural complex symplectic form which is nondegenerate and closed.

Remark 3.1. If the action of S^1 on M_r is free, then the quotient space M_r/S^1 is a complex manifold with a natural holomorphic symplectic structure. Unfortunately, in our case the action is not free since some points of $S[r]$ have nontrivial cyclic stabilizers.

Instead of $S[r]$, let us take a direct product X of r-copies of the surface S. In the same way, the preimage X_0 of $0 \in T^3$ will have a free action of S^1. The quotient variety $N_R = X_0/S^1$ is a complex symplectic variety, since the diagonal action of S^1 on X_0 is free in this case.

LEMMA 3.2. *Let $x \in M_r$ be a point with a nonzero stabilizer. Then the stabilizer is a cyclic group Z_l where l divides r and the set of Z_l-invariant points is a union of $B_l \subset M_r$, $dim_R B_l = 4r/l - 3$.*

Proof. The group $Z_l \in S^1$ stabilizes the scheme $x \in S[r]$ of length r provided the latter consists of translations by Z_l of the scheme of length r/l. These translations are parametrized by a variety $S[r/l]$ with a real dimension $4r/l$. The restriction of the projection p_T gives rise to a fibering of $S[r/l]$ over T^3. Hence the subvariety $B_l \subset S[r/l]$ has real dimension $4r/l - 3$.

Remark 3.3. Since $S[r/l]$ is smooth, all the components of B_l are smooth subvarieties of M_r. The map of p_t^l: $S[r/l] \to T^3$ is a combination of p_t for r/l and a multiplication by l on T^3. Hence B_l is in fact isomorphic to the union of l^3 copies of $M_{r/l}$.

COROLLARY 3.4. *Since $codim_R B_l$ in M_r is strictly greater than 2 for any l, $\pi_1(M_r) = \pi_1(M_r - \bigcup_l B_l)$. From the exact homotopy sequence for the fibration of $S[r]$ over T^3 we obtain that $\pi_1(M_r) = Z_m$ and is equal to the torsion subgroup in $H_1(S)$.*

Remark 3.5. The generator of $\pi_1(M_r)$ can be represented by the smallest orbit S^1/Z_r under the diagonal action of S^1. Since the fundamental group is abelian, we can consider its elements as oriented circles without fixing an initial point.

The generator coincides with the image of the composition of coordinate imbedding of S^1 into $S^r S^1$ and into $S[r]$. The space $S^r S^1$ is a fibration over S^1 with a ball D^{r-1} as a fiber. The projection p_s on S^1 arises from the group structure on S^1. The orbit S^1/Z_r projects isomorphically on S^1 and hence represents the same class e as the coordinate circle. The free orbit of S^1 represents the class re.

Notation. We shall denote as $\tilde{M}$ the open subvariety $M_r - \bigcup_l B_l$.

The action of S^1 on $\tilde{M}$ is free, and we denote the quotient $\tilde{M}/S^1$ by $\tilde{R}$. It is a smooth open submanifold of R_r with a complex symplectic structure induced by w_M.

LEMMA 3.6. $\pi_1(\tilde{R}) = Z_r$.

Proof. Indeed, there is a surjective map of $\pi_1(\tilde{M}) = Z_m$ onto $\pi_1(\tilde{R})$ and the kernel of it is generated by the class re. Hence $\pi_1(\tilde{R}) = Z_m/re = Z_r$.

COROLLARY 3.7. *There exists a nonramified Z_r-covering R' of $\tilde{R}$ which is an open simply connected symplectic complex manifold.*

We now have to show that there is a natural smooth compacitification Q of R' which is a ramified covering of R_r.

Consider now the structure of M_r and W_r in the neighborhood of a nonfree orbit. Let $(S^1/Z_l)_x$ be a nonfree orbit of S^1 in M_r and U_x be its small S^1-invariant neighborhood in M_r. Let D_x be a transversal ball to $(S^1/Z_l)_x$ in M_r. The action of S^1 on U_x induces a natural map q_x: $D_x \times S^1 \to U_x$. This map is a nonramified cyclic Z_l-covering of U_x.

The ball D_x has a natural complex symplectic structure induced by the form w, since the tangent field e_s to the orbits of S^1 coincides with the kernel of w on M_r. This structure is invariant under the action of Z_l on D_x. Therefore, D_x/Z_l has a natural structure of local complex space.

The above complex structures are compatible on the intersection of the quotients for different S^1-orbits in M_r since they correspond to the same form w.

Thus, R_r has a structure of complex variety, with D_x/Z_l being a neighborhood of a point.

The ball D_x is also transversal to the orbits of E_l in the space U_x^L which is the E_l-orbit of U_x in W_r. If G is a stabilizer in E_L on the E_L-orbit of $(S^1/Z_l)_x$ then $Z_l \subset G$ and G is a finite abelian group with a complex action on D_x. The local complex structure of W_r/E_L coincides with D_x/G.

LEMMA 3.8. *The projection map $p_R\colon\ R_r \to W_r/E_L$ is a finite map of compact complex spaces of degree r.*

Proof. We have to check that p_R is complex locally, but p_r coincides locally with a map $D_x/Z_l \to D_x/G$ where G is a finite group of complex transformations of D_x and $Z_l \subset G$. The map p_R is surjective since M_r intersects all E_L-orbits in W_L. Let S_0^1 be a complementary subgroup to S^1 in E^L. We have a natural action of S_0^1 on W_r/S^1. There is also a natural projection $p_s\colon\ W_r/S^1 \to S_0^1, p_s^{-1}(0) = R_r$.

The generic orbit of the diagonal action of S_0^1 on W_r/S^1 maps with the degree r into S_0^1. Therefore, the degree of $p_R\colon\ R_r \to W_r/E_L$ is equal to r.

Remark 3.9. The map p_R is nonramified over the open subvariety $\tilde{M} \subset M_r$ corresponding to the free orbits of E_L in W_r.

In order to show the existence of natural ramified r-covering of R_r we have to prove that the map $p_x\colon\ D_x \to D_x/Z_l$ can be constructed from D_x/Z_l in a canonical way.

LEMMA 3.10. *Suppose that nonfree orbits of Z_l in D_x constitute a complex subset F of codimension more than 1. Then:*

(1) *F/Z_l coincides with the set of singular points Sing $\subset D_x/Z_l$.*

(2) *$\pi_1(D_x/Z_l - Sing) = Z_l$.*

(3) *D_x coincides with holomorphic envelope of $D_x - F$.*

Proof. Indeed, (2) and (3) follow from the codimension condition for F, and (1) follows from the fact that the action of Z_l is topologically linear near x.

COROLLARY 3.11. *The subset F in our case corresponds to the quotient $\bigcup_l B_l/S^1$ and has a complex codimension > 1. Therefore, the map p_x is canonically defined by D_x/Z_l itself. In particular the maps p_{x_i} are compatible on the intersection of D_{x_i}/Z_{l_i} for different x_i, l_i.*

LEMMA 3.12. *The intersection of $\tilde{R}$ and D_x/Z_l is equal to $D_x/Z_l - Sing$. The cyclic universal covering R' of $\tilde{R}$ induces the universal covering $D_x - F \to D_x/Z_l - Sing$.*

Proof. According to the construction the covering $p_x\colon\ D_x \times S^1 \to U_x$ is induced from the universal Z_l covering of $D_x/Z_l - Sing$. Therefore, $\pi_1(D_x/Z_l - Sing)$ is generated by the image of a perturbed orbit $(S^1/Z_l)_x$ in U_x.

In a similar way the group $\pi_1(\tilde{R})$ is generated by a perturbed orbit S^1/Z_r and the map $\pi_1(D_x/Z_l - Sing) \to \pi(\tilde{R})$ is a monomorphism, $Z_l \to Z_r$. Hence, the Z_r-cyclic covering R' induces a Z_l-covering $D_x - F \to D_x/Z_l - Sing$.

We can consider now a variety Q obtained as a Z_r-cyclic ramified covering of R_r. In order to do this we complete the local neighborhood $D_x - F$ in R' as D_x. This completion is canonical and the completions are compatible on the open parts of the intersections of $D_{x_i} - F$ for different x_i. Therefore we obtain a smooth complex manifold Q as a ramified cyclic Z_r-covering of R_r.

LEMMA 3.13. *The manifold Q is simply connected and has a natural complex symplectic structure.*

Proof. Indeed, R' is simply connected and the real codimension of its complement in Q is greater than 1. Therefore Q is also simply connected.

The form w induces a complex symplectic structure on R' and on any of the complex balls D_x. Since the symplectic structures induced on these varieties are compatible we obtain the holomorphic nondegenerate (2.0)-form ω on Q.

Thus, we have constructed a compact complex symplectic simply connected manifold Q.

4. Absence of Kahler structure. Let us notice first that Q has a holomorphic projection on W_r/E_L of degree r^2. Thus it is exactly the manifold constructed by Guan [5].

Guan gave a proof that Q does not admit a Kahler structure, but I would like to present here a more geometrical argument. Namely, it turns out that Q contains surfaces which are non-Kahler and the structure of complex submanifolds in Q is interesting enough to be considered.

Let X be a product of r copies of S and $p_{X,E}\colon X \to E$ be a natural projection. Define N_E as a $p_{X,E}^{-1}(0)$. It is a complex manifold with a diagonal action of E_L and a holomorphic (2,0)-form w' obtained as a restriction of the holomorphic (2,0)-form on X invariant under symmetric group. The vector field tangent to the orbits of E_L coincides with the kernel of w' and the action of E_L on N_E is free. Therefore, the quotient variety $N = N_E/E_L$ has a nondegenerate holomorphic (2,0)-form ω'.

LEMMA 4.1. *There is a natural meromorphic projection $p_N\colon N \to W_r/E_L$.*

Proof. Indeed, there is a proper finite map of N to the variety V_r/E_L, where V_r is a preimage of $0 \in E$ under a natural projection $p'_T\colon S^r S \to E$. On the other hand, V_r/E_L and W_r/E_L are naturally bimeromorphically equivalent, since the equivalences between $S^r S$ and $S[r]$, V_r and W_r respectively are E_L-equivariant.

Remark 4.2. The manifold N_R constructed in the previous section is a cyclic nonramified Z_r-covering of N. This follows from the argument of lemma 3.8.

LEMMA 4.3. *There is a structure of toric fibration on N with the following properties:*

(1) *Its fiber T is isomorphic to a product of $r-1$ copies of the curve E_L.*

(2) *Its base is a torus T_0 isomorphic to a product of $r-1$ copies of the curve E_0.*

(3) *The fibration is described by $r-1$ integer vectors $v_i = (-m, -m, \ldots, 0, -m, \ldots, 0)$, where 0 stands on ith and $r-1$th places.*

Proof. The inital variety X is a fibration over the product T_E of r copies of the curve E. The fiber T' is a product of r copies of E_L and X is a principal T' fibration. It can be described by a diagonal scalar matrix $c_1^{i,j} = 0$ for $i \neq j$ and $c_1^{i,i} = m$. The variety $N \subset X$ is a fibration with T' as a fiber. The base $T_0 \subset T_E$ is also a product of $r-1$ elliptic curves E_0. The same holds for N/E_L with $T = T'/E_L$ as a fiber. The quotient $T = T'/E_L$ is isomorphic to the product of $r-1$ copies of E_0. The only problem now is how to make the choice of the representation of T as a product. We choose first $r-1$ coordinates in T_E as coordinates in T.

We can also choose the basis of elliptic curves E_i in T_0 as $(0, \ldots, x, 0, \ldots -x)_i$, where $x \in E_0$ and it is nontrivial for the ith and rth coordinates in the presentation of T_0 in the standard product T_E of r-copies of E_0.

The fibration of N is described by Chern classes $c_1^{i,j}$ in these coordinates. Here every series $c_1^{i,j}, j \in (1, \ldots, r-1)$ describes the restriction of T-fibration over the elliptic curve E_i corresponding to all the other coordinates but x_i being constant.

The fibration T' is described by the series $(0, \ldots, m, \ldots, m)_i$ and the quotient N with $T = T'/E_L$ as a fiber corresponds to $(-m, -m, \ldots, 0, -m, \ldots 0)_i$.

Elliptic curves parallel to E_i are parametrized by the points $t \in T_0/E_i$. Denote the preimage of $E_{i,t} \subset T_0$ in N as N_t.

LEMMA 4.4. *If $r > 2$ and $t \in T_0/E_i$ is a torsion point then N_t has a finite nonramified covering isomorphic to a product of Kodaira surface and $r-2$ copies of E_L.*

Proof. Indeed, the restriction of L_j to $E_{i,t}$ is equivalent to the translation of L_i by the jth coordinate of t. Therefore, if t is a torsion point the restriction of bundles L_j generate a subgoup of rank 1 in $PicE_{i,t}$. Hence we can apply Lemma 1.1.

Remark 4.5. We can similarly describe Kodaira surfaces in the preimage of any elliptic curve in T_0.

LEMMA 4.6. *If $r \geq 3$ then the manifold N contains Kodaira surfaces and the latter constitute a dense subset in N.*

Proof. Since torsion points constitute a dense subset in T/E_i, the set of nonintersecting Kodaira surfaces is dense in N due to the previous lemma.

COROLLARY 4.7. *The manifold Q contains a surface V which has a meromorphically surjective map onto some Kodaira surface. Indeed, we can take a Kodaira surface S in N passing through a point $x \in N$ such that the map of S into W_L/E_L is a meromorphic imbedding and $p_N(x)$ is well defined and is a smooth point of W_L/E_L. We can also assume that $p_N(x)$ lies outside of the ramification locus of the projection of Q onto W_l/E_l. We can take as V the preimage of $p_N(S)$ in Q.*

LEMMA 4.8. *The manifold Q does not admit a Kahler structure.*

Proof. It contains a surface V (may be singular) which has a finite meromorphic surjective map on S' by the above construction.

Now the lemma follows from the following general statement.

LEMMA 4.9. *The smooth complex manifold M does not admit a Kahler structure if it contains a singular compact complex surface V with a meromorphic surjection onto a Kodaira surface.*

Proof. Indeed if V is nonsingular then it is non-Kahler and therefore M cannot be Kahler. If V is singular then we can desingularize V by a sequence of blow ups on M with smooth centers. But if M was Kahler then the resulting variety M' obatined from M by a sequence of blow ups also must be Kahler. On the other hand, it contains a nonsingular model of V which is not Kahler. This yields a contradiction.

To be more precise, it is sufficient to blow up points and smooth curves on M in order to desingularize V. It follows from a general theorem of H. Hironaka, but in our case it is easy to make the blow up procedure explicit.

COROLLARY 4.10. *The variety Q constructed in Section 3 is a simply connected complex symplectic manifold which does not admit a Kahler metric.*

Remark 4.11. In fact, we do not need to deal with a complicated process of resolving singularities in our particular case.

By taking a generic enough Kodaira surface $S \subset N$ we can achieve that the image $S' \subset W_r/E_L$ will be a nonsingular surface. The surface S' is a blowing up of S at one point. We can also assume that S' does not intersect the ramification locus of the projection $Q \to W_r/E_L$. Thus we can actually construct a nonsingular surface $V \subset Q$ with a surjective map on the Kodaira surface S.

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